OBSERVATIONAL TESTS OF DIRAC'S COSMOLOGY

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Abstract

In this paper we submit Dirac's cosmology to three cosmological tests: the magnitude vs. red shift for optical galaxies and QSC. the metric angular diameter vs. red-shifts for radio-sources and QSO; and finally the isophotal angular diameters vs. z for optical galaxies. In each case a comparison is made not only with the observational data but also with the best fitted Friedmanians curves. Evolutionary effects are also included wherever necessary. While the m vs. z and the θ_i vs. z cannot be used to support or refute Dirac's cosmology, it is found that the θ_m vs. z is improved, even without evolutionary effects of the type introduced by De Young for radio-galaxies. A clean test between ordinary cosmology and Dirac's cosmology is proposed, based upon the behavior of θ_i/θ_m vs. z. This test was originally proposed by Sandage as a way of discriminating between ordinary cosmology and the tired light model. Such tests will be possible only when resolutions of the order of .1 arcsec are achieved with the Large Space Telescope. In the case of ordinary cosmology, $\theta_i/\theta_m \approx (1 + z)^{-2}$; whereas in Dirac's case θ_i/θ_m is independent of z, if the galaxy is treated as a polytrope of order 4. Such radically different behavior is a welcome feature of the theory, since it makes the test very hopeful.

For a certain choice of the parameters entering the theory, Dirac's results appear similar to those provided by the tired light model of Hubble and Tolman. A deeper analysis however, shows that the similarity is very superficial, Dirac's theory having the internal consistency of being able to define every parameter characterizing it in terms of observable quantities.

Difficulties with Dirac's theory discussed in another paper are taken up again here and a balance sheet subject to present day limitations is drawn, with the result that Dirac cosmology cannot be excluded as a viable alternative.

I. INTRODUCTION

The most elaborate description of the Universe as a whole is the one derivable from Einstein's theory. After one has accepted the Robertson-Walker (RW) metric representing a homogeneous and isotropic Universe, Einstein's equations provide the necessary machinery to set up a differential equation for the scale parameter of the RW metric, R(t). Three types of geometries are allowed, represented by $k = 0$, $+1$, -1. The first and second derivative of R(t), known as the Hubble constant and the deceleration parameter, are the cosmological parameters par excellence. They are defined by the following relations (Mc Vittie, 1964)

$$H_o = \frac{\dot{R}_o}{R_o} \quad , \quad q_o = - \frac{\ddot{R}_o \, R_o}{\dot{R}_o^2} \tag{1}$$

$$kc^2 = H_o^2 \, R_o^2 \, (2 \, q_o - 1) + \Lambda \, R_o^2 \, c^2 \tag{2}$$

where

$$\Lambda c^2 = 3 H_o^2 \, (\sigma_o - q_o) \quad , \tag{3}$$

$$\sigma_o = \frac{4 \pi \, G}{3 \, H_o^2} \, \rho_o \equiv \frac{1}{2} \, \frac{\rho_o}{\rho_c} \tag{4}$$

Λ is known as the cosmological constant. We have in general three parameters

$$H_o, \, q_o, \, \Lambda, \tag{5}$$

H_o and q_o, representing the left-hand side of Einstein's equations, are geometrical quantities giving the slope and curvature of the expansion parameter. ρ_o, the present-day matter density, represents the right-hand side of Einstein's equations, i.e. matter.

In their present form, Einstein's equations do not put any constraints upon H_o, q_o and ρ_o, except that they must be related via (1)-(4).

If $\Lambda = 0$, the previous relations shrink considerably and we are left with H_o and q_o. Clearly, once q_o is determined, the sign of k follows, as does the type of curvature of space, Equation (2). Equation (4) will not tell us anything that Equation (2) has not already told us, i.e., whether the Universe is open or closed. Equation (4), however, is more transparent in that it can be rewritten as

$$2 q_o = \frac{8 \pi G}{3 H_o^2} \rho_o = \frac{P.E.}{K.E.} \tag{6}$$

since

$$K.E. = \tfrac{1}{2} M v^2 = \tfrac{1}{2} M H_o^2 R_o^2$$

$$\tag{7}$$

$$P.E. = \frac{GM^2}{R_o} = \frac{4\pi}{3} R_o^2 GM \rho_o$$

If $2q_o > 1$, $k > 0$, then P.E. > K.E., i.e. the potential energy exceeds the kinetic energy and contraction will set in. For $2q_o < 1$, we have an open Universe.

This approach is attractively simple, since the overall question of openness or closedness seems to be amenable to the determination of two parameters, or more

strictly to one, q_0, since that alone determines the sign of k. Simple though it mi· t sound, the previous program has defied solution. Since 1960, Sandage has done the most extensive work in this subject and, as he himself has stressed, perhaps the only answer is that the data exclude the steady state. The numerical value of q_0 has fluctuated from a value centered around one (back in the sixties), to much smaller values ~ .03 in 1975. The physical implications are evidently diametrically different. (Sandage 1961; 1968; 1972; 1974, Sandage and Tammann 1975). Over the years the value of q_0 has steadily decreased toward zero. Sandage's work has not indicated whether this is the bottom value, or whether one could pass zero and approach negative q_0, an accelerating Universe. This is the latest proposal from an analysis that includes one more feature however, the evolution of the galaxies themselves (see IV).

After years of analysis of the classical cosmological tests, (i.e., the m vs. z and the (isophotal) angular-diameter vs. z relations), (Sandage 1962-1975) we are left with the unsavory taste of not having a definite answer for the value of q_0. The indeterminacy on q_0 raises fundamental questions as to whether the theory is incomplete or the observational test have left out some important factor.

Tinsley (1968; 1970; 1972; 1973) and more recently Gunn and Tinsley (1975) have upheld the point of view that the theoretical framework need not be changed, but that evolutionary effects must be included in the analysis. We shall discuss this point in IV. We can just quote here that one of the latest results of their work indicates that $q_0 < 0$, i.e. the Universe is accelerating. The cosmological constant, which for many years has undergone undeserved neglect, is again advocated and the full analysis is very complex.

While such an "observational approach" toward q_0 must certainly be pursued, it is clear that a "theoretical approach" is equally possible.

The steady state was one such possibility. It was conceived not as an alternative to Einstein's theory, but as a way to choose among too many possibilities by postulating the perfect cosmological principle. That immediately fixes a value for $q_o (= -1)$, and one parameter is eliminated from the basket. H_o becomes a universal constant and the cosmological diaspora is greatly reduced. It is now believed however, that the $3°K$ black-body supports the big bang and not the steady state theory.

The count of radio sources, although not in agreement with any cosmological model, is nevertheless interpreted as indicating the existence of evolutionary effects, which are excluded by fiat in the steady state. The situation is in a way analogous to the one described in the discussion of the value of q_o. We have not learned to discriminate among possible evolutionary models, but to disbelieve the steady state, or at least this is the position of many cosmologists.

Dirac's theory is another possibility. As we shall see, the postulates of the theory unequivocally fix the cosmological model, $q_o = 0$, leaving to other predictions the burden of a direct comparison with observational data. Dirac's theory was motivated by the desire to explain the existence of very large dimensionless numbers. It was not conceived as an alternative to Einstein's theory. It is based on a postulate of rather bold nature, which, however, has an ample predictability power, and is therefore amenable to observational test. The theory has gone through several vicissitudes mainly due to its original version, which has now been revised and considerably improved. Although from time to time there appear critics of the theory, so far as the author is aware, none of them has yet come up with a clear indication that the theory in its present form is in flagrant contradiction with clearly accepted observational facts.

Canuto and Lodenquai (1975) have discussed the main features of Dirac's theory, as well as several tests concerning the evolution of the Sun, white dwarfs, the unique $\dot{P}/P$ of the pulsar JP1953 and the source count arguments. In every case it was found that either Dirac's theory provides too small an effect to be observable, or in other cases (like in the pulsar case) it could be just the right explanation. However, in no instance was it found that the theory is unequivocably contradicting observed data.

The unacceptability of Dirac's theory exposed for example by Towe (1975) is based in our opinion on a misinterpretation of the behavior of atomic distances with cosmological time. A clear balance of pros and cons has not yet been drawn and any conclusion is premature.

In this paper we shall test the theory on three classical cosmological tests. The m, z relation for galaxies and quasars, the (θ_m, z) and (θ_i, z) relations for radio and optical galaxies. While the m, z relation is essentially left unchanged, the θ_m (metric diameter) vs. z relation for radio galaxies and QSO is improved. Furthermore, we shall show that the ratio between isophotal and angular diameter θ_i/θ_m, which in ordinary cosmology is independent of q_o and is given by

$$\theta_i/\theta_m = (1+z)^{-4/p} \quad , \qquad p \approx 2 \tag{8}$$

in Dirac's case is changed to

$$\theta_i/\theta_m = (1+z)^{-4/p + 2\omega/p} \tag{9}$$

the index ω being related to the polytropic index n of the galaxy by the relation

$$\omega = \frac{n-2}{n-3}$$

For a polytropic index n = 4, ϵ_i / ϵ_m is independent of z, a strikingly different prediction than the Big Bang (8). We consider this as the most stringent test of Dirac's theory.

Unfortunately this test cannot be made today since it requires metric diameters of about .13", a resolution not achievable with ground base optical astronomy. When the Large Space Telescope will be flown, the test will be possible, the resolution being only diffraction limited and therefore capable of differentiating between (8) and (9). The chief reason for the difference between (8) and (9) is the hypothesis of the continuous creation of matter and the decrease of the gravitational constant. The test proposed here would therefore be a confirmation of whether new matter (and so also photons) is continuously created in the Universe or not.

II. DIRAC COSMOLOGY

Dirac cosmology has been reviewed rather lengthly in a paper by Canuto and Lodenquai (1975, referred to as CL) and we shall refer the reader to that paper for a full presentation. We shall only state here the pertinent results.

The fundamental hypothesis is that the various so-called large numbers owe their large size to the fact that they depend on the age of the Universe. This is the only assumption of Dirac's theory. The rest follows in a consequential and almost unavoidable manner. Since $G \sim t^{-1}$, Dirac proposes the use of two metrics, the so-called atomic

metric, referred to which e, $\hbar$, m do not vary with time but $G \sim t^{-1}$, and the Einstein metric, in which Einstein's equations are written. In that metric G, M (mass of object are constant like in the ordinary Einstein theory, but the atomic constants e, m, $\hbar$, etc. vary with time.

In Table 1 we summarize the results of such variations. Here t is the age of the Universe in atomic units. From Table 1 we can easily deduce that the number of photons in a monochromatic beam should increase in time. In fact, considering that the total energy of the beam

$$E_T = \hbar \nu \, N_\gamma \quad , \tag{10}$$

must be constant in Einstein units and $\qquad$ that $h_E \sim t^{-3}$ and $\nu_E \sim 1/\lambda_E \sim$ const, we derive

$$N_\gamma \sim t^3 \tag{11}$$

Since N_γ is a pure number, we must expect the same behavior in atomic units, where, upon remembering that $h \sim$ const, $\nu \sim \lambda^{-1} \sim t^{-1}$, it will then follow that $E_T \sim t^2$. We shall write the exponent in Equation (11) as α and specify the numerical value only at the end. As explained in CL, in atomic units, time intervals get stretched in the amount

$$(\delta t)_A \sim t(\delta t)_E \tag{12}$$

TABLE I

Time Dependence

	Atomic Units	Einstein Units
v, (velocity)	t^0	t^0
e, (charge)	t^0	$t^{-3/2}$
m, (mass)	t^0	t^{-2}
h, (Planck's const.)	t^0	t^{-3}
G, (grav. const.)	t^{-1}	t^0
M, (bulk mass)	t^2	t^0
r, (distance)	t	t^0
λ, (wavelength)	t	t^0
h^2/me^2, (Bohr radius)	t^0	t^{-1}

where $(\delta t)_A$ is the lapse of time taken for a given phenomenon to occur when the age of the Universe was t. As the Universe ages, that same physical phenomenon will take a longer time to occur. The relation between the Einstein and atomic metrics is therefore given by

$$ds_A \sim t \, ds_E \qquad (13)$$

Since in Einstein units Dirac's cosmology requires the use of a static Universe i.e., the original Einstein Universe, we shall write for ds_E^2 (Tolman, 1966)

$$ds_E^2 = d\tau^2 - \left(\frac{d\rho^2}{1 - \rho^2/R_E^2} + \rho^2 \, d\Omega^2 \right) \qquad (14)$$

From now on we shall use the greek letters τ and ρ to indicate times and distances in Einstein Units. In Equation (14), R_E is a constant defined by the equations

$$\Lambda - 8\pi p = R_E^{-2}$$

$$\Lambda + 8\pi \rho = 3 \, R_E^{-2} \qquad (15)$$

$$4\pi \, (\rho + p/c^2) = \frac{c^2}{G \, R_E^2}$$

Finally we must remember that the definition of red-shift is

$$1 + z = \frac{t_o}{t} \tag{16}$$

where t_o is the age of the present age of the Universe in atomic units and t the time of emission of the photon under consideration.

III. THE m vs. z RELATION

Let us consider a photon that leaves a source with energy $h\nu_e$. If N_e photons are emitted in the time interval Δt_e, the absolute luminosity of the source at the time of emission is

$$\mathcal{L}(t_e) = \frac{h\nu_e}{\Delta t_e} N_e$$

The average flow of energy per unit of time recorded by the observer at the time t_o, is

$$\ell(t_o) = \frac{h\nu_o}{\Delta t_o} \frac{N_o}{S} \tag{17}$$

where S is the area of the pseudo-sphere surrounding the observer. From Table 1 and Eq. (12) we deduce that (in atomic units)

$$h\nu_o = h\nu_e \left(\frac{t_e}{t_o} \right) , \quad \Delta t_e = \left(\frac{t_e}{t_o} \right) \Delta t_o \tag{18}$$

since $\nu = c/\lambda \cong c/t$, for any time t. Therefore

$$\mathcal{L}(t_o) = \mathcal{L}(t_e) \left(\frac{t_e}{t_o} \right)^2 \left(\frac{N_o}{N_e} \right) \frac{1}{S} \tag{19}$$

From (16) we finally have

$$\mathcal{L}(t_o) = \frac{\mathcal{L}(t_e)}{(1+z)^2 S} \left(\frac{N_o}{N_e} \right) \tag{20}$$

The factor $(1+z)^2$ also enters into the usual derivation of $\mathcal{L}(t_o)$, the only difference being that in Eq. (19) one has $R(t_e)/R(t_o)$ instead of t_e/t_o. The final result is however the same. The difference lies in the factor N_o/N_e. In the ordinary theory, where there is no creation of photons along the way, the number of photons that left the source is clearly the same as the one that reaches the observer and so $N_e = N_o$. (Mc Vittie, 1964 page 164). In Dirac's theory this is no longer true. N_o is greater than N_e, since the original photons have multiplied themselves on the way to the observer. Using (11) we must write (we leave the exponent undetermined to be able to recover the old result)

$$N_o = N_e \left(\frac{t_o}{t_e} \right)^\alpha$$

or

$$N_o = N_e (1+z)^\alpha \tag{21}$$

so that finally

$$\mathcal{L}(t_o) = \frac{\mathcal{L}(t_e)}{(1+z)^2 S} (1+z)^\alpha \qquad (22)$$

Since the surface S is proportional to

$$S = 4\pi \, r_e^2 \, R^2(t_o) \quad ,$$

Equation (22) gives

$$\mathcal{L}(t_o) = \mathcal{L}(t_e) \, \frac{(1+z)^\alpha}{4\pi \, (1+z)^2 \, r_e^2 \, R^2(t_o)} \qquad (23)$$

In ordinary cosmology as well as in Dirac theory (when working in atomic units) the radial trajectory of a photon is determined by putting $ds^2 = 0$ in the metric

$$ds^2 = c^2 \, dt^2 - R^2(t) \left[\frac{dr^2}{1 - k \, r^2} + r^2 \, d\Omega^2 \right] \qquad (24)$$

thus obtaining

$$\int_{t_e}^{t_o} \frac{c\,dt}{R(t)} = \int_o^{r_e} \frac{dr}{\sqrt{1-k\,r^2}}$$

Since the right-hand side can be integrated for any value of k with the result

$$(-k)^{-1/2} \sinh^{-1} \left[(-k)^{1/2} r_e \right]$$

we have that the general expression for r_e is

$$r_e = (-k)^{-1/2} \sinh \left\{ (-k)^{1/2} \int_{t_e}^{t_o} \frac{c\,dt}{R(t)} \right\} \tag{25}$$

At this point the two theories differ. In fact, in ordinary cosmology one uses Einstein equations to express R(t) as a function of t and the parameters q_o, Λ, σ_o etc. The result of such analysis is (Solheim 1966, Appendix II)

$$\Lambda = o: \qquad (1+z)\,r_e = F(z, q)$$

$$\tag{26}$$

$$F(z, q) = q^{-2} \left[qz + (q-1) \left\{ \sqrt{1 + 2qz} - 1 \right\} \right]$$

with

$$F(z, 1) = z$$

$$F(z, 0) = z(1 + z/2) \tag{27}$$

$$F(z, q) = z\left[1 - \tfrac{1}{2}(q - 1)z\right] \qquad \text{small } z's$$

Substitution of (26) into (23) with $\alpha = o$, yields the following result for ordinary cosmology, $H_o R(t_o) = c$,

$$\ell_N(t_o) = \left(\frac{H_o}{c}\right)^2 \frac{\mathcal{L}_N(t_e)}{4\pi \, F^2 \, (z, \, q)} \tag{28}$$

Within the framework of Dirac theory, we cannot use Einstein's equations to express $R(t)$ vs t. However, as we have explained in deriving Equation (29) of LC, the theory itself provides us with the function $R(t)$, namely

$$R(t) = R_E \left(\frac{t}{t_o}\right) \tag{29}$$

so that finally we have from (25)

$$r_e(b, \, z) = \begin{cases} \sin\left[b\ell n\,(1 + z)\right] \, , & k = +1 & \text{(30a)} \\[2mm] b\,\ell n\,(1 + z) \, , & k = 0 & \text{(30b)} \\[2mm] \sinh\left[b\,\ell n\,(1 + z)\right] \, , & k = -1 & \text{(30c)} \end{cases}$$

where the parameter b is defined as

$$b = \frac{c}{H_o R_E} \quad (31)$$

or upon using Einstein equations to express R_E in terms of ρ via (15) with $p = o$

$$b^2 = \frac{c^2}{H_o^2 R_E^2} = \frac{4\pi G \rho}{H_o^2} = \frac{3}{2} \frac{\rho}{\rho_c} \quad (32)$$

where the critical density ρ_c is defined as

$$\rho_c = \frac{3 H_o^2}{8\pi G} = 4.697 \; 10^{-30} \left(\frac{H_o}{50}\right)^2 g. \; cm^{-3}$$

The parameter k must be chosen +1. In fact, inserting (29) into (24), calling $r \, R_E = \rho$ and comparing the result so obtained with (14), shows that (13) is satisfied only if $k = +1$.

In conclusion, within Dirac's theory the apparent luminosity is

$$\ell_D(t_o) = \left(\frac{H_o}{c}\right)^2 \frac{\mathscr{L}_D(t_e) (1+z)^\alpha b^2}{4\pi [(1+z) r_e (b, z)]^2} \quad (33)$$

We have explicitly written $\mathcal{L}_N(t_e)$ and $\mathcal{L}_D(t_e)$ for the total absolute luminosities in ordinary cosmology and Dirac cosmology in order to emphasize that they are different.

Before translating (28) and (33) into magnitudes, and discuss the (m, z) relation, we would like to point out several interesting points. From what we said before, one could get the impression that within Dirac theory, Einstein equations are actually not used since we have arrived at our final result, (33) without them. This is actually not so. The parameter connecting the two theories is R_E in (29). In fact only through the use of Einstein equations (15), were we able to give a physical interpretation to R_E, Equation (32). In a way this is to be expected. In fact in ordinary cosmology the formula for r_e contains q_o which in turn is written as (see (3) with $\Lambda = o$)

$$2 q_o = \rho/\rho_c$$

in such a way that a fit to the observed luminosities (or magnitudes) can be interpreted as a way of determining the density ρ (in units of ρ_c). The aim of the (m, z) test is to determine q_o. In Dirac theory we do not have a q_o and therefore the last equation correcting q_o and ρ does not exist. The physics is however the same. Through the use of (32), the parameter b is again connected with the density and therefore the m vs z test has the same purpose of determining ρ/ρ_c. Finally, we would like to notice that Equation (30a) is the same as the one given by the tired light model, first proposed by Hubble and Tolman (1935). In that model however, there is no way to relate b to the density and therefore no equation (32). b was an entirely free parameter and that makes the tired light model a less complete structure than Dirac's theory.

IV. THE ABSOLUTE LUMINOSITIES - EVOLUTIONARY EFFECTS

In the work of Sandage throughout the years it was assumed that the absolute luminosity of a given galaxy does not change in time and therefore

$$\mathcal{L}_N(t_e) \equiv \mathcal{L}_N(t_o) \tag{34}$$

This is equivalent to saying that the absolute luminosity of a galaxy is not affected by the change in luminosity that its components undergo during the transit time of the light from the galaxy to us.

Recently this assumption has been rediscussed and it seems that evolutionary effects can seriously affect our knowledge of q_o. We shall return to this point in the following. For the moment let us assume that (34) is valid. Can we assume that

$$\mathcal{L}_D(t_e) = \mathcal{L}_D(t_o) \tag{35}$$

is also valid? The answer is no. In fact, even without stellar evolutionary effects, Eq. (35) cannot hold in Dirac's theory since the mass of every star increases with time like t^2 and this alone must have some effect. In order to study the effect we first need the general relation giving us R, T and L vs. M and G. Using the full set of equations of stellar structure and assuming a perfect gas equation of state and expressing the nuclear energy generation and the opacity as

$$\epsilon = \epsilon_o \, \rho \, T^n \quad , \quad k = k_o \, \rho^{k_1} \, T^{k_2} \tag{36}$$

the following results can be derived for the radius R, temperature T, and intrinsic luminosity L of each star,

$$R \sim G^{g_1} \, M^{m_1} \qquad \sim t^{2 m_1 - g_1} \tag{37}$$

$$T \sim G^{1 - g_1} \, M^{1 - m_1} \qquad \sim t^{1 + g_1 - 2 m_1} \tag{38}$$

$$L \sim G^{\gamma} \, M^{\delta} \qquad \sim t^{2\delta - \gamma} \tag{39}$$

where

$$g_1 = \frac{n + k_2 - 4}{n + 3 + 3 k_1 + k_2} \quad , \quad m_1 = \frac{n - 1 + k_1 + k_2}{n + 3 + 3 k_1 + k_2} \tag{40}$$

$$\gamma \equiv \frac{4 n + 3 k_1 n - 3 k_2 + 12}{n + 3 + 3 k_1 + k_2} \quad , \quad \delta \equiv \frac{3 n + 2 n k_1 + 3 k_1 - k_2 + 9}{n + 3 + 3 k_1 + k_2} \tag{41}$$

The particular case of Kramers opacity $k_1 = 1$, $k_2 = -7/2$ was first worked out by Ganiow (1967). Since in Dirac's theory $G \sim t^{-1}$, $M \sim t^2$, the luminosity L varies like

$$L \sim t^{2\delta - \gamma} \quad , \quad 2\gamma - \delta = \frac{(2 + k_1) n + 6 k_1 + k_2 + 6}{n + 3 + 3 k_1 + k_2} \tag{42}$$

In general n is large with respect to both k_1 and k_2 and so the quantity $2\delta - \gamma$ can be approximated by

$$2\delta - \gamma \simeq 2 + k_1 \tag{43}$$

so that (39) can be written as

$$L_D(t) = \left(\frac{t}{t_o} \right)^{2\delta - \gamma} L_N(t) \tag{44}$$

$$L_D(t) = \frac{L_N(t)}{(1 + z)^{2\delta - \gamma}} \approx \frac{L_N(t)}{(1 + z)^3} \tag{45}$$

A quick insertion of (45) into (33) with $\alpha = 3$ shows that the effect of photon multiplication is almost exactly canceled by the lowest intrinsic luminosity the source had at the time of emission.

In order to evaluate the total luminosities $\mathcal{L}_N$ and $\mathcal{L}_D$, we shall generalize a model employed by Tinsley (1972b), in which all the stars in gE galaxies were formed at the time $t = o$ with a mass function

$$dN/dM = N_o M^{-x} \tag{46}$$

with x independent of time.

Using (39) we then have

$$\frac{dN}{dL} = \frac{dN}{dM}\frac{dM}{dL} \sim N_o \, G^{\gamma(x-1)/\delta} \, L^{(1-\delta-x)/\delta} \tag{47}$$

so that the total luminosity $\mathcal{L}_D(t)$ at the time t when the main-sequence turn off luminosities is $L_D(t)$ is given by

$$\mathcal{L}_D(t) = \int_o^{L_D(t)} L\, dN(L) \sim N_o \, G^{\gamma(x-1)/\delta} \, L_D(t)^{1+(1-x)/\delta} \tag{48}$$

Let us first extract the Dirac dependence by using $G \sim t^{-1}$ and (39). The result is

$$\mathcal{L}_D(t) = \mathcal{L}_N(t) \, (t/t_o)^{2\delta-\gamma} \tag{49}$$

$$= \mathcal{L}_N(t) \, (1+z)^{\gamma-2\delta} \tag{50}$$

Equation (50) proves that (45) is valid also for the total luminosity. In going from (48) to (49) we have eliminated N_o in favor of N, the total number of stars being a time-independent quantity in both cosmological models (barring exoteric effects like cannibalism, recently advocated by Ostriker and Tremaine 1975).

Considering that the time spent on the MS is

$$t \simeq \frac{M}{L} \sim L^{(1-\delta)/\delta} \tag{51}$$

we can write

$$L_N(t) \sim \left(\frac{t}{t_o}\right)^{-\frac{\delta}{\delta-1}} \tag{52}$$

For the total luminosity we can again use (48) except that now we can drop the index D and the G dependence. We then obtain[*)]

$$\mathcal{L}_N(t) = \mathcal{L}_N(t_o) \left(\frac{t}{t_o}\right)^{-k} = \mathcal{L}_N(t_o) (1+z)^k \tag{53}$$

$$k = \frac{\delta+1-x}{\delta-1} \tag{54}$$

On the basis of Tinsley (1972b) result that the giants and all stars beyond the MS contribute very little to evolution, we shall limit ourselves to (53).

[*)] We think that there cannot be any possibility of confusion of this evolutionary index k with the geometrical k used in (24), since that was set equal to one and never used below Equation (33).

Substituting now (53) into (28) gives

$$\iota_N(t_o) = \left(\frac{H_o}{c} \right)^2 \frac{\mathcal{L}_N(t_o)}{4\pi \, F^{*2}(z, q)} \tag{55}$$

where

$$F^*(z, q) = F(z, q) (1 + z)^{-k/2} \tag{56}$$

For small z's we have, using the last of (27)

$$F^*(z, q) = F(z, q) (1 - \tfrac{1}{2} kz) = F(z, q^*) \tag{57}$$

$$q^* = q + k \tag{58}$$

which is just a renormalization of q. This means that the m, z relation actually determines q^* not q and since k is positive, the geometrically meaningful parameter (i.e. q) will always be smaller than the one we measure. Considering (53) as a Taylor expansion for small $t - t_o$, we can write

$$k = - \frac{1}{H_o} \left(\frac{\dot{\mathcal{L}}}{\mathcal{L}} \right)_o = \frac{.92}{H_o} \frac{dM}{dt} = 18.51 \left(\frac{50}{H_o} \right) \frac{dM}{dt_9} \tag{59}$$

A change of .03 magnitudes per billion years can lower the value of q^* by .5. This rather large disturbance of the real value of q has recently generated a confused situation. In fact, should the actual value of q determined from the (m, z) relation, i.e. q^*, turn out to be close to zero, as the latest determinations seem to indicate, then the geometrically meaningful q would be negative, implying an accelerating Universe. This is possible only if one includes the cosmological constant, after which the situation becomes almost arbitrarily complex. On the contrary, evolutionary effects are very easy to account for in the Dirac cosmology, as we shall see.

Substituting (53) into (50) and then into (33), gives for the Dirac's case

$$\ell_D(t_o) = \left(\frac{H_o}{c}\right)^2 \frac{\mathcal{L}_N(t_o)\ b^2}{4\pi\ G^2\ (z,\ b)} \tag{60}$$

where

$$G(z,\ b) = (1+z)\ (1+z)^{-(\alpha+\gamma-2\delta)/2}\ (1+z)^{-k/2}\ r_e\ (z,\ b) \tag{61}$$

Translating luminosities into magnitudes, yields the results

$$m_N = 5\ \lg\ cF^*\ (z,\ q) + C_N \tag{62}$$

$$C_N = M + 5\ \lg\ \left(\frac{50}{H}\right) + 16.504 \tag{63}$$

$$m_D = 5 \lg c \, G(z, b) + C_D(b) \quad . \tag{64}$$

$$C_D(b) = C_N - 5 \lg b \tag{65}$$

where we have used (31) to express R_E in terms of H_o and b.

In Table 2 we present m_N vs. z as well as m_D vs. z for three different values of the density parameter ρ/ρ_c and for $k = 0$ (no evolutionary correction) and $k = 1$. For the case of ordinary cosmology, we employed Sandage's best fit (1972c)

$$m_N = 5 \lg c \, z - 6.46 \tag{66}$$

which is a particular case of (62) when

$$q^* = 1 \; , \; C_N = -6.46 \tag{67}$$

In Dirac's case we have taken $\alpha + \gamma - 2\delta \approx 0$ as from (40) and (41). In order to have a feeling for the results provided by Dirac's theory, we have plotted in Figure 1 the observational data, as from the work of Sandage (1972d), Sandage's fit to the points (solid curve) corresponding to Equation (66) and the two extreme cases in Dirac's theory. The curve bending towards the left corresponds to column 9, i.e. to

$$\rho/\rho_c = 3 \; , \; k = 1 \; , \; x = 2 \tag{68}$$

TABLE 2. APPARENT MAGNITUDES VS. z FOR ORDINARY COSMOLOGY m_N AND DIRAC'S COSMOLOGY m_D, FOR THREE DENSITIES ρ/ρ_c; $k = 0$ CORRESPONDS TO NO-EVOLUTIONARY EFFECTS.

z	lg cz	m_N ($q' = 1$)	$m_D^{(k=0)}$			$m_D^{(k=1)}$		
			$\rho/\rho_c = 10^{-2}$	$\rho/\rho_c = 1$	$\rho/\rho_c = 3$	$\rho/\rho_c = 10^{-2}$	$\rho/\rho_c = 1$	$\rho/\rho_c = 3$
.053	4.2	14.54	14.60	14.60	14.59	14.55	14.54	14.54
.084	4.4	15.54	15.63	15.63	15.62	15.55	15.54	15.53
.132	4.6	16.54	16.66	16.65	16.64	16.53	16.51	16.50
.21	4.8	17.54	17.74	17.72	17.68	17.53	17.51	17.47
.33	5.0	18.54	18.82	18.77	18.68	18.51	18.46	18.37
.528	5.2	19.54	19.98	19.88	19.68	19.52	19.42	19.22
.81	5.4	20.54	21.17	20.97	20.53	20.52	20.31	19.86
1.32	5.6	21.54	22.37	21.98	21.07	21.46	21.06	20.15
2.10	5.8	22.54	23.64	22.00	20.89	22.41	21.67	19.67

whereas the one to the right of Sandage's curve corresponds to column 4, i.e. to

$$\rho/\rho_c = .01 \quad , \quad k = 0 \tag{69}$$

The remaining results listed in Table 2 fall between the two extreme cases.

V. METRIC ANGULAR DIAMETERS

Let us consider two events occurring at the points (Mc Vittie 1964)

$$A(r_e, \theta_e, \varphi_e) \quad , \quad B(r_e, \theta_e, + \Delta \theta_e, \varphi_e)$$

at the same time t_e and let the observer be located at (0, 0, 0) at the time t_o. The two emission events are separated by a local distance

$$\Re = r_e R(t_e) \Delta \theta \tag{70}$$

where we have used the metric (24). The "metric angular diameter" of the source is defined as

$$\theta_m = \frac{\Re}{r_e R(t_e)} = y \frac{(1 + z)}{r_e(z)} = y \frac{(1 + z)^2}{F(z, q)} \tag{71}$$

$$y = \frac{1}{c} H_o \Re = 17.15 \left(\frac{H_o}{50} \right) \left(\frac{\Re(kpc)}{500} \right) \quad (arcsec)$$

where $\Re$ is the radius of the source and where we have used the $R(t_o) = R(t) (1 + z)$. As usual, $r_e(z)$ is defined by equation (26).

In the case of Dirac's cosmology we have

$$\theta_m = \frac{\Re}{r_e \, R(t_e)} = y \, \frac{(1 + z)b}{r_e(z, \, b)} \tag{72}$$

when we have used (29) to eliminate $R(t_e)$ and (31); $r_e(z, b)$ is given by (30a). It's easy to understand that the factor b should appear in the numerator of (72). In fact for small values of q's (or equivalently of the density), the function $r_e(z, q)$ of (71) goes to z, whereas $r_e(z, b)$ goes to zb. The b in the numerator just makes (72) finite and independent of b for small values of b's.

In the case of ordinary cosmology the results for

$$q = +1, \, -1, \, 0, \, 1/2$$

are as follows

$$\theta_m = y \, (1 + z)^2 \, z^{-1} \, (1 + z)^{-1} \qquad q = 0$$

$$\theta_m = y \, (1 + z)^2 \, z^{-1} \qquad q = 1$$

$$\theta_m = y \, z^{-1} \, (1 + z) \qquad q = -1$$

$$\theta_m = \frac{1}{2} y (1 + z)^2 \{1 + z - (1 + z)^{1/2}\}^{-1} \qquad q = 1/2$$

$$\theta_m = y z^{-1} \qquad\qquad\qquad \text{(Eucl. model)} \qquad (73)$$

In the case of optical galaxies, the use of metric diameters has been of limited application, mainly because what we measure are isophotal rather than metric diameters.

In Figure 2 of Baum (1972), the (θ_m, z) relation is presented for several values of q_o. The famous feature of a minimum in the (θ_m, z) curve that led Hoyle to stress the possible cosmological importance of this test, has so far not been of great help, both because of the reason just stated and also because of the difficulty in recording galaxies' images at large redshifts, where the differences among several world models become important.

The situation is very different in the case of radio-sources. The most typical morphological feature of a radio-source is the double structure, i.e. the existence for two bright lobes on the side of a central galaxy or QSO. The angular separation between the two components is indeed a metric diameter.

Several plots of θ_m vs. z for radio galaxies have been made in the past (Legg 1970; Miley 1971; Wardle and Miley 1974). The results of Wardle and Miley are reported in Figure 2, where the data for 166 QSO are collected. Curves I and II correspond to

    i) Static Euclidean Model

    ii) $q = 1/2$

Before we explain curve III, we want to present the case for Dirac cosmology.

In order to write (72) in the framework of Dirac cosmology, it is not enough to simply use r_e as given by (30a). The quantity y (or more precisely the radius) can in fact vary with time. If we consider the galaxy to be a polytrope of index n

$$p \sim \rho^\gamma \quad , \quad \gamma = 1 + 1/n$$

the variables G, M and $\Re$ must satisfy the relation

$$G \, M^{2 - \gamma} \, \Re^{3\gamma - 4} = const$$

as shown in Equation (81) of LC. Since $G \sim t^{-1}$, $M \sim t^2$ the radius R must vary as

$$\Re \sim t^\omega \quad , \quad \omega = \frac{n - 2}{n - 3} \tag{74}$$

On the grounds of Equation (16) we can write

$$\Re(z) = \Re(0) \, (1 + z)^{-\omega} \tag{75}$$

so that the final result in the Dirac case is (in arcsec)

$$\theta_m = 17.15 \, \frac{b(1 + z)^{1 - \omega}}{r_e(z, b)} \left(\frac{\Re}{500} \right) \left(\frac{H_o}{50} \right) \tag{76}$$

where $\Re$ is given in units of 500 kpc, and H_o in units of 50 km sec^{-1} Mpc^{-1}. Curves I and II in Figure 2 are for $\Re = 500$ and $H_o = 50$.

This however is not the full story. In fact, recent work on radio-galaxies suggest that the radius $\Re$ can change with time. It is an evolutionary effect of an altogether different nature than the one described before. This effect is entirely due to the dynamics of the galaxy or rather of the two radio lobes that in the expansion perform work against the intergalactic medium. DeYoung (1971) has performed numerical calculations of this effect and his result is that the radius $\Re$ varies like

$$\Re (z) = \Re (o) (1 + z)^{-4/5} \tag{77}$$

so that

$$y = \frac{1}{c} H_o \Re (o) (1 + z)^{-4/5} \tag{78}$$

When this is substituted in the expression for θ_m $(q = 1/2)$ in (71), the result is curve III in Figure 2.

The variation of $\Re$ vs. z, (75), implied by Dirac's theory has however nothing to do with (78) and is in no way a substitute for it. As always, the effects of Dirac's theory must be superimposed or added to any other already existing physical effect. If we believe DeYoung's model, we must include (77) in (76).

In Table 3 we present the values of θ_m for the flat-Euclidean case (second column) as well as for Dirac's cosmology for $\omega = 0, 1, 2$, for three values of the density parameter ρ/ρ_c, namely

TABLE 3

METRIC ANGULAR DIAMETERS (ARCSECS)

H = 50 , ℜ = 500

z	θ_m Eucl. Mod.	$\theta_m(w=2)$ 1	2	3	$\theta_m(w=1)$ 1	2	3	$\theta_m(w=0)$ 1	2	3
.02	875.5	819.1	819.2	819.3	866.1	866.2	866.3	888	883	886
.04	428.7	420.5	420.6	420.9	437	437	437.8	454	455	455
.08	214.4	206.3	206.6	207.3	223	223	223.8	241	241	242
.1	171.5	163.6	164.	165	180	180	181	198	198	199
.2	85.7	78.4	79	83.4	94	95	96.4	113	114	116
.4	42.9	36.1	37.5	39.7	51	52.4	55.6	71.4	73.4	77.8
.6	28.6	22.8	24.1	27.1	36	38.6	43.3	58.4	61.7	69.3
.8	21.4	16.2	17.7	21.3	29.2	31.9	38.4	52.6	57.3	69.1
1.0	17.1	12.1	13.9	18.3	25	27.9	36.6	49.5	56	73.1
1.2	14.3	9.9	11.6	16.6	21.8	25.5	36.6	48	56.2	80.4
1.6	10.7	6.9	8.8	15.6	18	22.8	40	47	59.3	105
1.8	9.5	5.9	7.9	16.0	16.7	22.0	44.5	47	61.7	124
2.0	8.6	5.2	7.2	16.7	15.7	21.5	50	47	64.6	150
2.1	7.1	4.14	6.2	20.6	14.1	21	70	48	71.6	238
2.6	6.6	3.7	5.8	24.5	13.4	21	88.4	48.4	75.6	318
2.8	6.1	3.4	5.5	31.4	12.9	21	119.4	49	80	454

(1) ρ/ρc = .01 , (2) ρ/ρc = 1 ; (3) ρ/ρc = 3 .

$$(1) \quad \frac{\rho}{\rho_c} = \frac{1}{100} \quad , \quad (2) \quad \frac{\rho}{\rho_c} = 1 \quad (3) \quad \frac{\rho}{\rho_c} = 3$$

In Figure 2 we present the data of Wardle and Miley and five theoretical curves. Curves I and II correspond to the flat-Euclidean model and $q = 1/2$. Curve III incorporates the DeYoung correction (77) for the $q = 1/2$ case. Curve D correspond to Dirac's case

$$\rho/\rho_o = 0.01 \quad , \quad n = 4 \quad , \quad \omega = 2 \quad ,$$

third column of Table 3. The curve marked DDY is column 3 of Table 3 multiplied by the DeYoung correction factor $(1 + z)^{-4/5}$. Among all the Dirac cases listed in Table 3 we have chosen to plot the one corresponding to a polytrope of order 4. The density is 100 times smaller than the critical density. Inspection of Table 3 reveals that only for the high-density Universes ($\rho/\rho_c \geq 1$) the θ_m vs. z curve has a minimum. The lower density Universe (for $\omega = 0, 1, 2$) do not exhibit such a feature.

VI. ISOPHOTAL DIAMETERS

Let us now study the isophotal diameters, i.e. the angles actually measured in optical astronomy. When one observes a close-by galaxy, the largest isophotal diameter coincides with the metric diameter, defined as the actual geometrical size from one side to the other. When the same galaxy is placed further away, the real boundary of the galaxy are lost, being dimmer than the integral parts. It will therefore be very difficult to measure metric diameters and only isophotal diameters will be measurable. The maximum (measurable) isophotal diameter will no longer coincide with the metric

diameter, but it will represent just a diameter slightly larger than the brightest central part. Evidently the farther away the galaxy, the smaller will be the maximum isophotal diameter compared to the actual metric diameter. In conclusion, one can expect that

$$\varepsilon_m > \varepsilon_i$$

In order to derive the form of ε_i we must derive the surface brightness, defined as (Sandage 1972a)

$$B \simeq \iota \, \varepsilon_m^{-2} \tag{79}$$

where ι is the apparent luminosity, Equations (28) and (33). In the case of ordinary cosmology, the result is

$$B \simeq \frac{\mathcal{L}_N(t_e)}{y^2} \, (1 + z)^{-4} \tag{80}$$

where it is seen that the quantity r_e, that depends upon q, has actually disappeared. Equation (80) is therefore valid for any value of q. We have written $\mathcal{L}_N(t_e)$ and y explicitly since evolutionary effects can come in and bring an extra dependence on z. However since we shall deal only with optical galaxies, the radius $y = \Re \, H_o/c$ is not expected to change and we can therefore drop it from now on. $\mathcal{L}_N(t_e)$ can however vary.

In the case of Dirac cosmology as a result of Equations (76), and (33) we have

$$B_D \simeq \mathcal{L}_D(t_e) \, (1 + z)^{-4 + \alpha + 2\omega}$$

Converting $\mathcal{L}_D$ into $\mathcal{L}_N$ via (47), we finally obtain

$$B_D \simeq \mathcal{L}_N(t_e) \, (1 + z)^{-4 + (\alpha + \gamma - 25) + 2\omega} \tag{81}$$

We are now in a position to compute θ_i, the isophotal angle. The determination is usually made by using a (semi-empirical) formula of Hubble giving the variation of B vs. θ, i.e.

$$B_i = B_o \, (1 + \theta_i/\theta_m)^{-p} \tag{82}$$

where the index p is about 2. B_o is the central brightness and for $B_o >> B_i$ we can write

$$\frac{\theta_i}{\theta_m} \simeq B_o^{1/p} \tag{83}$$

Clearly B_i is of no interest since it is a quantity decided upon by the observer, who sets the luminosity he wants to observe. B_o is however an intrinsic property and it varies with z as dictated by Eq. (80) or (81).

We shall have

$$\frac{\varepsilon_i}{\varepsilon_m} \sim \mathcal{L}_N^{1/p} (t_e) \, (1 + z)^{-4/p} \tag{84}$$

(Dirac) $\quad \dfrac{\varepsilon_i}{\varepsilon_m} \sim \mathcal{L}_N^{1/p} (t_e) \, (1 + z)^{-4/p + (\alpha + \gamma - 2\delta)/p + 2\omega/p}$ $\qquad$ (85)

For the case considered before

$$\alpha + \gamma - 2\delta \approx 0$$

we have

$$\frac{\theta_i}{\theta_m} \sim \mathcal{L}_N^{1/p} (t_e) \, (1 + z)^{-4/p}$$

$$\tag{86}$$

(Dirac) $\quad \dfrac{\theta_i}{\theta_m} \sim \mathcal{L}_N^{1/p} (t_e) \, (1 + z)^{-4/p + 2\omega/p}$

Specifically for the Dirac case we have

$$\frac{\theta_i}{\theta_m} \sim \quad
\begin{array}{lll}
1) & \text{independent of } z & \text{(polytrope order 4)} \\[2ex]
2) & (1 + z)^{-1/p} & \text{(polytrope order 5)} \\[2ex]
3) & (1 + z)^{-2/p} & \text{(polytrope order } \infty)
\end{array}
\tag{87}$$

It is interesting to note that the polytrope of order 5 yields the same result as the tired light model.

Sandage (1974d) has proposed that one of the most useful experiments achievable with the large space telescope (LST) could be precisely that of measuring metric diameters at say $z = .5$ or greater, and then comparing the results with the predictions of both Big Bang and the tired light cosmologies.

We would like to extend such a proposal as a powerful test of Dirac's theory. Difficult evolutionary effects have been subtracted off, since they are the same in both theories. Unless ω is zero, Dirac predictions will differ from ordinary cosmology. It seems to be a very clean observational test unless it so happens that θ_i / θ_m falls off exactly like $(1 + z)^{-1/p}$, in which case we'll have no way to know whether Dirac's theory or the tired light model is correct.

The LST has the capability of resolving less than .1 arcsec and if it is ever flown, it will certainly allow for this most interesting cosmological test. Let us now study the isophotal diameter. Eliminating θ_m from Equations (84) and (85) via (71) and (76) we get

$$\theta_i \sim \mathfrak{L}_N^{1/p} (t_e) \; \frac{(1 + z)^{2 - 4/p}}{F^* (z, q)} \tag{88}$$

$$\theta_i \sim \mathfrak{L}_N^{1/p} (t_e) \; \frac{b}{(1 + z)^\nu \cdot r_e (z, b)} \tag{89}$$

$$\nu = 4/p - 1 + \omega (1 - 2/p) - (\alpha + \gamma - 2\delta)/p \tag{90}$$

For the Hubble value $p = 2$, ν is independent of z and its value is $1.$ We can therefore write

$$\text{(Dirac)} \qquad \varepsilon_i = \mathcal{L}_N^{1/p} (t_e) \; \frac{b}{(1 + z) \, r_e \, (z, \, b)} \tag{91}$$

In Figure 3, we present the observational points as from the work of Sandage (1972), as well as his fit using (88)

$$\lg \theta_i = - .986 \lg cz + 5.331 \tag{92}$$

which corresponds to

$$p = 2 \; , \qquad q^* = 1 \; , \qquad \mathcal{L}_N(t_e) = \mathcal{L}_N(t_o) = \text{const.}$$

In the case of Dirac's cosmology, we have instead

$$\lg \theta_i = - .986 \lg c(1 + z) \, r_e \, (z, \, b) - .986 \lg b + 5.331 \tag{93}$$

corresponding to (91). The normalization constants has been chosen in such a way that (92) and (93) coincide for $z \to o$. In Table 4, we present (92) and (93) for several values of z, for the three values of ρ/ρ_c employed before.

TABLE 4. ISOPHOTAL ANGULAR DIAMETERS (ARCSEC) FOR ORDINARY COSMOLOGY AND DIRAC'S COSMOLOGY

	lg cz	lg $\theta_i(q^* = 1)$	lg θ_i (Dirac)		
			(1)	(2)	(3)
.003	3.0	2.37	2.42	2.42	2.42
.0105	3.5	1.88	1.88	1.88	1.88
.0333	4.0	1.38	1.38	1.38	1.38
.1054	4.5	.894	.87	.87	.87
.333	5.0	.401	.34	.35	.37

(1) $\rho/\rho_c = 10^{-2}$, (2) $\rho/\rho_c = 1$, (3) $\rho/\rho_c = 3$

Concluding Remarks

A viable alternative to existing cosmological theories must perform at least equally well and possibly improve the fit to the traditional tests m vs. z, ε_m vs. z and ϱ_i vs. z. One of the aims of this paper was to show that this requirement is indeed satisfied by Dirac's theory. In fact inspection of Figures 1, 2, 3 indicates that the theory has certainly passed the test. Moreover it has the nonmarginal property of having improved the ϱ_m vs. z test.

Let us first analyze the m vs. z. Used extensively by Sandage, this test has over the years provided a value of q_o much greater than the one obtained from the observed amount of deuterium, if this is of cosmological origin. This last test indicates an open Universe $q_o \cong .03$, a value never achieved by the m vs. z analysis, if evolutionary effects are not included.

Recently, Sandage and Tammann (1975) have concluded that despite their earlier belief in the use of the m vs. z relation as a cosmological test, the following relation should instead be used

$$t_o = H_o^{-1} f(q_o) \; , \tag{94}$$

once the Hubble constant has been determined. Even without knowing the exact age of the Universe t_o, the very fact that it must be greater than the age of globular clusters, poses restrictions on q_o. With the most recent value of $H_o (\sim 50 \pm 3)$, and an age of globular clusters around $14 \, 10^9$ yr, q_o cannot exceed $\approx .03$. We shall recall in fact that

$$f(q_o) \leq 1 \quad , \quad f(o) = 1 \tag{95}$$

for any positive q_o.

Such a value of q_o is now in accord with the one obtained from the abundance of deuterium. To reach such an agreement however we had to set aside the m vs. z relation.

Tinsley and Gunn, by retaining the m vs. z relation and including evolutionary effects have shown that the value of q_o can indeed be lowered. No matter which of the two approaches will turn out to be more reliable, there is little doubt that we are converging toward a value of q_o much smaller than one.

Dirac's theory demands $q_o = o$ and this is clearly in harmony with all we have said so far. The spirit of Dirac cosmology is different however. In Friedmanian cosmologies, the main parameter q_o, defined in terms of the scale factor R(t) alone, is coupled via Einstein equations to the amount of matter in the Universe,

$$2q_o = \rho/\rho_c$$

The m vs. z relation as well as the knowledge of the amount of deuterium yields a value for q_o, i.e. they determine the type of geometry of the Universe.

In Dirac's theory, the geometry of space is determined by the very postulates of the theory.

Einstein equations (as such) can only be written in Einstein units and the choice of geometry is unique, the Universe must be static. For that we need a cosmological constant Λ, whose value cannot however be determined by Dirac's theory. From equation 15 we have

$$\Lambda \simeq R_E^{-2} = \frac{4\pi G \rho}{c^2} \tag{96}$$

In performing any of the previous tests, we have employed quantities measured in atomic units. In these units we do not have Einstein equations. However if the space is homogeneous and isotropic, the metric is of the RW type, Equation (24). The expansion factor R(t) has a time behavior dictated again by the postulates of the theory, i.e. we have

$$R(t) \sim t \quad , \quad k = 1 \tag{97}$$

and this fixes the type of space $q_o = o$. Having determined the function R(t) a priori, it could appear as though the previous tests have lost their main objective. This is not the case. In fact the proportionality constant between R(t) and t, is precisely R_E, which is given in terms of the density.

The search for a fit of the theoretical m, θ_m, θ_i to the observed values is therefore a search for the amount of matter in the Universe. From Figure 1 one could be tempted to conclude that Dirac's theory performs no better than the Friedmanian cosmologies. Actually we think it does better. In fact, we limit ourselves to non-evolutionary models, the value $q \approx 1$ provided by the best fit is simply too high compared to .03 quoted before. In order to reduce it, we need evolutionary

effects that can however make it negative. In Dirac's theory evolutionary effects can be accounted for without any extreme consequences.

What about the ϱ_m vs. z relation? Ordinary Friedmanian models do not fit the data exceedingly well, whereas the Euclidean curve seems to do better. Even with evolutionary effects of the De Young type, the situation is not greatly improved. Dirac's cosmology even without evolutionary effects provides already a better fit.

The ϱ_i vs. z relation is not greatly altered. Here, however, much of what we said concerning the m, z curve can be restated. In fact evolutionary effects count again very heavily and the apparent $q^* \leq 1$ fit actually could well correspond to negative q.

Finally, we would like to comment on the important prediction for ϱ_i/θ_m. As explained earlier, we attach great importance to this test since it can definitely rule in favor or against Dirac's cosmology in a way not matched by any of the other tests.

When the LST will be flown and the test hopefully carried out, the viability of Dirac's theory will definitely be checked.

ACKNOWLEDGMENTS

FIGURE CAPTIONS

Figure 1.   The apparent magnitude vs. red shift relation. The solid curve corre-
sponding to normal cosmology is from the work of Sandage (1972a). The
two extreme cases of Dirac's cosmology, corresponding to columns 9
and 4 of Table 2, are represented by the dotted lines.

Figure 2.   The largest metric angular diameter for radio sources. Curves I, II, III
correspond to ordinary cosmology, as from the work of Wardle and Miley
(1974), whereas D and DDY correspond to Dirac's cosmology. For de-
tails see the text.

Figure 3.   The isophotal diameter vs. z. The solid curves corresponding to $q^* = 2.5$,
1, -1 for ordinary cosmology are from Sandage (1972b). Dirac's cosmol-
ogy gives rise to the dotted curve.

REFERENCES

Baum, W. A., 1972, in External galaxies and quasi-stellar objects, ed. D. S. Evans,
[Holland: Reidell, p. 393.]

Canuto, V., and J. Lodenquai, 1975, Phys. Rev. D. (submitted).

De Young, D. S., 1971, Ap. J. $\underline{167}$, 541.

Gamow, G., 1967, Proc. Nat. Acad. Sciences $\underline{57}$, 187.

Gunn, J. E., and Tinsley, B. M. 1975, Nature $\underline{247}$, 454.

Legg, T. H., 1970, Nature, $\underline{226}$, 65.

Miley, G. K., 1971, M.N.R.A.S., $\underline{152}$, 477.

McVittie, G. C., 1964, General Relativity and Cosmology [London: Chapman and
Hall, Ltd.].

Ostriker, J. P., and S. D. Tremaine, 1975, Princeton preprint.

Sandage, A., 1961a, Ap. J. $\underline{133}$, 355.

_____., 1961b, Ap. J. $\underline{134}$, 916.

_____., 1968, Observatory, $\underline{88}$, 91.

_____., 1972a, Ap. J., $\underline{173}$, 485.

_____., 1972b, Ap. J., $\underline{178}$, 1.

_____., 1972c, Ap. J., $\underline{178}$, 25.

_____., 1972d, Q. Jl. R.A.S., $\underline{13}$, 282.

_____., 1974a, Ap. J., $\underline{190}$, 525.

_____., 1974b, Ap. J., $\underline{191}$, 603.

_____., 1974c, Ap. J., $\underline{194}$, 223.

_____., 1974d, in Large Space Telescope, A New Tool for Science, A.I.A.A.,
12th Aerospace Sciences Meeting, NASA report, p. 19.

_____., and Tamman, G. A. 1975, Ap. J. $\underline{196}$, 313.

Solheim, J. E., 1966, M.N.R.A.S. 133, 321.

Tinsley, B. M., 1968, Ap. J. 151, 547.

_____., 1970, Astroph. and Space Science 6, 344.

_____., 1972a, Ap. J. 178, L39.

_____., 1972b, Ap. J. 173, L93.

_____., 1973, Ap. J. 184, L41.

Tolmann, R. C., 1966, Relativity, Thermodynamics and Cosmology, [Oxford:

    Clarendon Press].

Towe, K. M., 1975, Nature, 257, 115.

Wardle, J. F. C., and G. K. Miley, 1974, Astron. & Astrophys. 30, 305.

Weinberg, S., 1973, Principles and Applications of the General Theory of Relativity,

    [New York: John Wiley].

Figure 1

Legend:
- ● RADIO GALAXIES
- △ RADIO QSO
- + QUIET QSO
- ⊕ RED QUIET QSO
- × SEYFERT (TOTAL)

Axes: $\log cz$ (vertical), V_c (horizontal)

⊕ B264

1

LARGEST ANGULAR SIZE (arcsec) vs REDSHIFT

I : Flat - Euclidean Model
II : q=1/2
III : q=1/2 + Evol. Corr.
D : Dirac
DDY : Dirac - DeYoung

2

ISOPHOTAL ANGULAR DIAMETERS

3